Quiz 4 Solutions

1. Compute $\iint_D 2xy\,dA$, where D is the triangular region with vertices $(0,0)$, $(1,2)$, and $(0,3)$.

   Solution.

$$
\begin{aligned}
\iint_D 2xy\,dA &= \int_0^1 \int_{2x}^{3-x} 2xy\,dy\,dx \\
&= \int_0^1 [xy^2]_{y=2x}^{y=3-x}\,dx \\
&= \int_0^1 (-3x^3 - 6x^2 + 9x)\,dx \\
&= \left[-\frac{3}{4}x^4 - 2x^3 + \frac{9}{2}x^2\right]_0^1 \\
&= \frac{7}{4}.
\end{aligned}
$$

2. Use triple integrals to find the volume of the tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$.

   Solution. Let A be the region obtained by projecting the volume V onto the xy-plane. Also, the plane $2x+y+z = 4$ intersects the xy-plane in $y = 4 - 2x$. Using vertical strips, the region A is the xy-plane can be described by $A = \{(x, y) \,|\, 0 \le x \le 2,\ 0 \le y \le 4 - 2x\}$.

   Therefore the volume V is given by

$$
\begin{aligned}
V &= \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} dz\,dy\,dx \\
&= \int_0^2 \int_0^{4-2x} (4 - 2x - y)\,dy\,dx \\
&= \int_0^2 [4y - 2xy - \frac{y^2}{2}]_{y=0}^{y=4-2x}\,dx \\
&= \int_0^2 (2x^2 - 8x + 8)\,dx \\
&= \left[\frac{2}{3}x^3 - 4x^2 + 8x\right]_0^2 \\
&= \frac{16}{3}.
\end{aligned}
$$